

Rotational stability of linear string baryon configuration

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Abstract

For the linear baryon string model with three massive points (three quarks) connected sequentially by the relativistic strings the initial-boundary value problem is stated and solved in general. This problem implies defining a classical motion of the system on the base of given initial position and initial velocities of string points. The given solution work reduces the initial-boundary value problem for the considered model to the system of ordinary differential equations that can be integrated numerically in general. The examples of numerical simulation of the system motion are considered, which, in particular, allow to answer to the old question about stability of a flat uniform rotation of the linear string configuration.

Introduction

String models of baryon differ from each other by geometric character of junction of three massive points by relativistic strings. Four variants are possible: a) the “three-string” model or Y-configuration with three strings from three quarks joined in the fourth massless point [1, 2]; b) the “triangle” model or Δ -configuration with pairwise connection of three quarks by three relativistic strings [6, 7]; c) the quark-diquark model q-qq [3] (from the point of view of classical dynamics it coincides with the meson model of relativistic string with massive ends [4, 5]); d) the linear configuration q-q-q with quarks connected in series [8].

In the present work the latter model is considered. It is not investigated practically in comparison with others. The common opinion was that the

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configuration q-q-q is unstable. It seemed obvious, that for the rotational motion of this system the centrifugal force rejects the middle quark at an end, and the system q-q-q transforms in quark-diquark q-qq [3]. However this conclusion remained only speculative till now — for its proof it was necessary to develop any way of solution of the initial-boundary value problem for the given model with arbitrary initial conditions. It would allow us to investigate on stability, for example, the known rotational motion, in which the rectilinear string uniformly rotates and the middle quark is at rest at the center of rotation.

In this work the initial-boundary value problem for classical motion of the linear configuration q-q-q is solved on the base of approach, applied earlier to the “meson” model of relativistic string with massive ends [10].

The initial-boundary value problem for the baryonic model has additional difficulties in comparison with the mesonic one. In particular, for an arbitrary motion of the q-q-q system the world surface of the string is not smooth on the trajectory the middle quark, moreover, there is no parametrization with the conformally flat induced metric and with the quark trajectories as coordinate lines.

These problems and also equations of motion and boundary conditions for the given model are considered in Sect. 1. For solving this initial-boundary value problem the Fourier method (widely used in the string theory [12]) is not applicable owing to nonlinearity of the boundary conditions and necessity of realization of the orthonormality conditions. Similar difficulties connected with the latter conditions arise in numerical solution of the problem with the help of difference schemes. In Sect. 2 we suggest another approach to solving the given problem. In section 3 with the help of numerical methods the concrete example of motion for the q-q-q string baryon model is investigated. Driving for.

1. Equations of motion and boundary conditions On quark trajectories

Let's consider an open relativistic string with the tension γ carrying three pointlike masses m_1, m_2, m_3 (the masses m_1 and m_3 are placed at the ends of the string). The action for this system is

$$S[X^\mu] = - \int_{\tau_1}^{\tau_2} d\tau \left\{ \gamma \int_{\sigma_1(\tau)}^{\sigma_3(\tau)} \left[\langle \dot{X}, X' \rangle^2 - \dot{X}^2 X'^2 \right]^{\frac{1}{2}} d\sigma + \sum_{i=1}^3 m_i \sqrt{X_i^{*2}(\tau)} \right\}. \quad (1)$$

Here $X^\mu(\tau, \sigma)$ are coordinates of a point of the string in D -dimensional Minkowski space $R^{1,D-1}$, the speed of light $c = 1$, $(\tau, \sigma) \in \Omega = \Omega_1 \cup \Omega_2$ (Fig. 1),

$\langle a, b \rangle = a^\mu b_\mu$ — (pseudo)scalar product, $\dot{X}^\mu = \partial_\tau X^\mu$, $X'^\mu = \partial_\sigma X^\mu$, $X_i^{*\mu}(\tau) = \frac{d}{d\tau} X^\mu(\tau, \sigma_i(\tau))$; $\sigma_i(\tau)$ ($i = 1, 2, 3$) — inner coordinates of world lines of point-like masses (quarks).

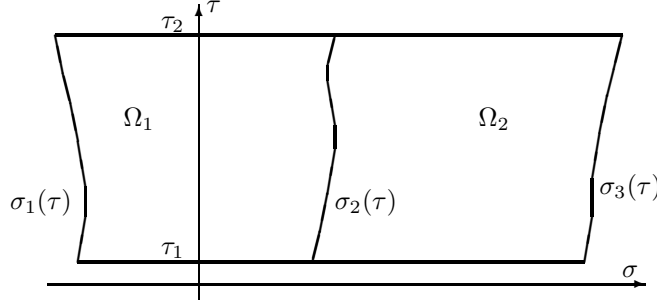


Figure 1: Domain of integration in (1).

We use the notations

$$L(\dot{X}, X') = -\gamma \sqrt{\langle \dot{X}, X' \rangle^2 - \dot{X}^2 X'^2}, \quad \Lambda_i(X_i^*) = -m_i \sqrt{X_i^{*2}}.$$

Varying action (1) and equating the variation $\delta S[X^\mu]$ to zero we shall receive the equations of motion

$$\frac{\partial}{\partial \tau} \frac{\partial L}{\partial \dot{X}^\mu} + \frac{\partial}{\partial \sigma} \frac{\partial L}{\partial X'^\mu} = 0, \quad (\tau, \sigma) \in \Omega. \quad (2)$$

$\sigma = \sigma_i(\tau)$. and boundary conditions at the quark trajectories $\sigma = \sigma_i(\tau)$.

To derive boundary conditions in the model “q-q-q” we are to take into account the discontinuities of \dot{X}^μ , X'^μ on the line $\sigma = \sigma_2(\tau)$. Thereby the term

$$\iint_{\Delta} \left[\frac{\partial}{\partial \tau} \left(\frac{\partial L}{\partial \dot{X}^\mu} \delta X^\mu \right) + \frac{\partial}{\partial \sigma} \left(\frac{\partial L}{\partial X'^\mu} \delta X^\mu \right) \right] d\tau d\sigma \quad (3)$$

in $\delta S[X^\mu]$ transformed using the Green’s formula equals the sum of two curvilinear integrals of internal boundary values along the borders of the domains Ω_1 , Ω_2 (Fig.1) and therefore — in the following boundary conditions:

$$\frac{d}{d\tau} \frac{\partial \Lambda_1}{\partial X_1^{*\mu}} - \left[\frac{\partial L}{\partial X'^\mu} - \frac{\partial L}{\partial \dot{X}^\mu} \sigma_1'(\tau) \right] \Big|_{\sigma=\sigma_1(\tau)} = 0,$$

$$\begin{aligned}
& \frac{d}{d\tau} \frac{\partial \Lambda_2}{\partial X_2^{*\mu}} - \left[\frac{\partial L}{\partial X'^\mu} - \frac{\partial L}{\partial \dot{X}^\mu} \sigma_2'(\tau) \right] \Big|_{\sigma=\sigma_2(\tau)+0} + \\
& + \left[\frac{\partial L}{\partial X'^\mu} - \frac{\partial L}{\partial \dot{X}^\mu} \sigma_2'(\tau) \right] \Big|_{\sigma=\sigma_2(\tau)-0} = 0, \\
& \frac{d}{d\tau} \frac{\partial \Lambda_3}{\partial X_3^{*\mu}} + \left[\frac{\partial L}{\partial X'^\mu} - \frac{\partial L}{\partial \dot{X}^\mu} \sigma_3'(\tau) \right] \Big|_{\sigma=\sigma_3(\tau)} = 0,
\end{aligned} \tag{4}$$

The equations of motion of the string and the boundary conditions have the simplest form if with the help of nondegenerate reparametrization $\tau = \tau(\tilde{\tau}, \tilde{\sigma})$, $\sigma = \sigma(\tilde{\tau}, \tilde{\sigma})$ the induced metric on the world surface of the string is made continuous and conformally-flat [6], i.e., satisfies the orthonormality conditions.

$$\dot{X}^2 + X'^2 = 0, \quad \langle \dot{X}, X' \rangle = 0. \tag{5}$$

Under conditions (5) the equations of motion (2) become linear

$$\ddot{X}^\mu - X''^\mu = 0 \tag{6}$$

and the boundary conditions (4) take the simplest form

$$\begin{aligned}
& m_1 \frac{d}{d\tau} U_1^\mu(\tau) - \gamma (X'^\mu + \sigma_1'(\tau) \dot{X}^\mu) \Big|_{\sigma=\sigma_1(\tau)} = 0, \\
& m_2 \frac{d}{d\tau} U_2^\mu(\tau) + \gamma (X'^\mu + \sigma_2'(\tau) \dot{X}^\mu) \Big|_{\sigma=\sigma_2(\tau)-0} - \\
& - \gamma (X'^\mu + \sigma_2'(\tau) \dot{X}^\mu) \Big|_{\sigma=\sigma_2(\tau)+0} = 0, \\
& m_3 \frac{d}{d\tau} U_3^\mu(\tau) + \gamma (X'^\mu + \sigma_3'(\tau) \dot{X}^\mu) \Big|_{\sigma=\sigma_3(\tau)} = 0.
\end{aligned} \tag{7}$$

Here the notations

$$U_i^\mu(\tau) = \frac{X_i^{*\mu}(\tau)}{\sqrt{X_i^{*2}(\tau)}} = \frac{\dot{X}^\mu + \sigma_i'(\tau) X'^\mu}{\sqrt{\dot{X}^2 \cdot (1 - \sigma_i'^2)}} \Big|_{\sigma=\sigma_i(\tau)}, \quad i = 1, 2, 3 \tag{8}$$

for unit $R^{1,D-1}$ -velocity vector of i -th quark was taken.

From the physical point of view equations (4) are the 2-nd Newtonian law for the material points m_i .

Eqs. (5) and (6) are invariant with respect to reparametrizations $\tau \pm \sigma = f_\pm(\tilde{\tau} \pm \tilde{\sigma})$ [5]. Choosing these two arbitrary functions f_\pm one can fix two (of three) functions $\sigma_i(\tau)$, for example, in the form

$$\sigma_1(\tau) = 0, \quad \sigma_3(\tau) = \pi. \tag{9}$$

The first equation (9) may be obtained at the first step by using the above reparametrization with required f_+ and $f_-(\eta) = \eta$, the second equation (9) — at the second step with $f_+ = f_-$. This procedure doesn't permit to fix $\sigma_2 = \text{const}$ for all τ in general.

For the initial-boundary value problem the function $\sigma_2(\tau)$ should be calculated on the base of initial data.

The function $X^\mu(\tau, \sigma)$ is continuous in Ω but on the line $\sigma_2(\tau)$ its derivatives can have discontinuities of 1-st type (except for tangential V_i^μ and $\frac{d}{d\tau}V_i^\mu$).

2. Initial-boundary value problem

The initial-boundary value problem for a relativistic string with the action (1) we shall formulate in general as follows: “to find a solution of the equation (6) $X^\mu(\tau, \sigma)$, $\mu = 0, 1, \dots, D-1$, smooth in the domain

$$\Omega = (\tau, \sigma) : \tau > \tau_0(\sigma); \quad \sigma_1(\tau) < \sigma < \sigma_3(\tau). \quad (10)$$

continuously differentiable on $\partial\Omega$ satisfying to the orthonormality conditions (5) and boundary conditions (7) and two given initial conditions: an initial position of the string in Minkowski space and an initial velocity of a point of the string”.

An initial position of the string can be given as a spacelike curve in Minkowski space

$$x^\mu = \rho^\mu(\lambda), \quad \lambda \in [\lambda_1, \lambda_3], \quad (11)$$

satisfying the conditions $[\rho'(\lambda)]^2 < 0$. Initial velocities on the initial curve can be given by timelike $R^{1,D-1}$ -vector $v^\mu(\lambda)$, $\lambda \in [\lambda_1, \lambda_3]$.

To solve the problem we set the initial curve (11) on the world surface of the string parametrically

$$\tau = \tau(\lambda), \quad \sigma = \sigma(\lambda), \quad \lambda \in [\lambda_1, \lambda_3]$$

(in expression (10) the same line is given as $\tau = \tau_0(\sigma)$).

In these notations the initial conditions (initial position of a string and the initial velocities) can be given in general [10, 11]

$$X^\mu(\tau(\lambda), \sigma(\lambda)) = \rho^\mu(\lambda), \quad \lambda_1 \leq \lambda \leq \lambda_3, \quad (12)$$

$$\alpha(\lambda) \dot{X}^\mu(\tau(\lambda), \sigma(\lambda)) + \beta(\lambda) X'^\mu(\tau(\lambda), \sigma(\lambda)) = \rho^\mu(\lambda), \quad \lambda_1 \leq \lambda \leq \lambda_3, \quad (13)$$

where $\alpha(\lambda)$, $\beta(\lambda)$ are arbitrary functions satisfying the inequality $\alpha(\lambda) > |\beta(\lambda)|$ and (if the relations (9) are tied) the condition on at the ends $\beta(\lambda_1) = \beta(\lambda_3) = 0$.

Functions $\tau(\lambda)$, $\sigma(\lambda)$, $\alpha(\lambda)$, $\beta(\lambda)$ are related by formulas [10]

$$\sigma'(\lambda) = \frac{\alpha\Delta + \beta P}{v^2}, \quad \tau'(\lambda) = \frac{\beta\Delta + \alpha P}{v^2}, \quad (14)$$

where $P(\lambda) = \langle v(\lambda), \rho'(\lambda) \rangle$, $\Delta(\lambda) = \sqrt{\langle v, \rho' \rangle^2 - v^2 \rho'^2}$. It is convenient to choose the constants of integration in (14) so that $\tau(\lambda_1) = \sigma(\lambda_1) = 0$ (Fig. 2). Realization of the condition $\sigma(\lambda_3) = \pi$ is possible by multiplying $\alpha(\lambda)$ and $\beta(\lambda)$ by a constant.

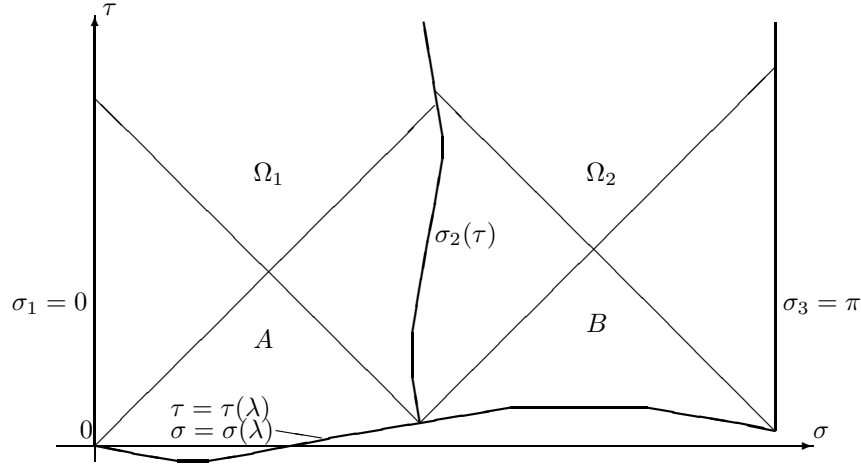


Figure 2: Ω .

The line $\sigma_2(\tau)$ divides domains Ω into two domains Ω_1 and Ω_2 . Let's consider the common solutions of the equation (6) in these domains

$$\begin{aligned} X^\mu(\tau, \sigma) &= \frac{1}{2} [\Psi_{1+}^\mu(\tau + \sigma) + \Psi_{1-}^\mu(\tau - \sigma)], \quad (\tau, \sigma) \in \Omega_1 \\ X^\mu(\tau, \sigma) &= \frac{1}{2} [\Psi_{2+}^\mu(\tau + \sigma) + \Psi_{2-}^\mu(\tau - \sigma)], \quad (\tau, \sigma) \in \Omega_2. \end{aligned} \quad (15)$$

We can find the functions $\Psi'_{i\pm}^\mu$, $i = 1, 2$ using the formulas [10]

$$\Psi'_{i\pm}^\mu(\tau(\lambda) \pm \sigma(\lambda)) = \frac{(\Delta \mp P)v^\mu \pm v^2 \rho'^\mu}{\Delta(\alpha(\lambda) \pm \beta(\lambda))} \quad (16)$$

If (as mentioned above) we choose

$$\tau(\lambda_1) = \sigma(\lambda_1) = 0, \quad \sigma(\lambda_3) = \pi,$$

that the formulas (16) will determine $\Psi_{i\pm}^\mu(\xi)$ in the following segments:

$$\begin{aligned} \Psi_{1+}^\mu(\xi), \xi \in [0, \tau(\lambda_2) + \sigma(\lambda_2)], \quad \Psi_{2+}^\mu(\xi), \xi \in [\tau(\lambda_2) + \sigma(\lambda_2), \tau(\lambda_3) + \pi], \\ \Psi_{1-}^\mu(\xi), \xi \in [\tau(\lambda_2) - \sigma(\lambda_2), 0], \quad \Psi_{2-}^\mu(\xi), \xi \in [\tau(\lambda_3) - \pi, \tau(\lambda_2) - \sigma(\lambda_2)], \end{aligned} \quad (17)$$

that allows us to find the solution of the problem as (15) in the zones $A = \{(\tau, \sigma) : \tau(\lambda(\sigma)) < \tau \leq \sigma; \tau + \sigma \leq \tau(\lambda_2) + \sigma(\lambda_2)\}$ $B = \{(\tau, \sigma) : \tau(\lambda(\sigma)) < \tau \leq \sigma + \tau(\lambda_2) - \sigma(\lambda_2); \tau + \sigma \leq \tau(\lambda_3) + \pi\}$ (Fig. 2). In these zones the solution depends only on initial data without influence of the boundaries. The constant of the integration in Eq. (16) is determined with the help of the initial condition (12).

In others zones of the domains Ω_1 and Ω_2 we shall Obtain the solution by prolongating the functions $\Psi_{i\pm}^\mu$ on all positive semiaxis with the help of the boundary conditions.

For the given initial-boundary value problem the boundary conditions look like (7). By choosing the inner equations of the string ends as (9) and by substituting in Eqs. (7) common solutions (15) we transform this system (it is supposed that the masses m_1, m_2, m_3 are final and nonzero):

$$\frac{dU_1^\mu(\tau)}{d\tau} - \frac{\gamma}{2m_1} [\Psi_{1+}'^\mu(\tau) - \Psi_{1-}'^\mu(\tau)] = 0, \quad (18)$$

$$\begin{aligned} \frac{dU_2^\mu(\tau)}{d\tau} + \frac{\gamma}{2m_2} \left[(1 + \sigma_2') [\Psi_{1+}'^\mu(\tau + \sigma_2) - \Psi_{2+}'^\mu(\tau + \sigma_2)] + \right. \\ \left. + (1 - \sigma_2') [\Psi_{2-}'^\mu(\tau - \sigma_2) - \Psi_{1-}'^\mu(\tau - \sigma_2)] \right] = 0, \end{aligned} \quad (19)$$

$$\frac{dU_3^\mu(\tau)}{d\tau} + \frac{\gamma}{2m_3} [\Psi_{2+}'^\mu(\tau + \pi) - \Psi_{2-}'^\mu(\tau - \pi)] = 0. \quad (20)$$

The expressions for the unit velocity vectors (8) after substitutions (9), (16) may be rewritten as

$$\begin{aligned} U_1^\mu(\tau) &= \frac{\Psi_{1+}'^\mu(\tau) + \Psi_{1-}'^\mu(\tau)}{\sqrt{2\langle \Psi_{1+}'^\mu(\tau), \Psi_{1-}'^\mu(\tau) \rangle}}, \quad U_3^\mu(\tau) = \frac{\Psi_{2+}'^\mu(\tau + \pi) + \Psi_{2-}'^\mu(\tau - \pi)}{\sqrt{2\langle \Psi_{2+}'^\mu(\tau + \pi), \Psi_{2-}'^\mu(\tau - \pi) \rangle}}, \\ U_2^\mu(\tau) &= \frac{(1 + \sigma_2') \Psi_{1+}'^\mu(\tau + \sigma_2) + (1 - \sigma_2') \Psi_{1-}'^\mu(\tau - \sigma_2)}{\sqrt{2(1 - \sigma_2'^2) \langle \Psi_{1+}'^\mu(\tau + \sigma_2), \Psi_{1-}'^\mu(\tau - \sigma_2) \rangle}} = \\ &= \frac{(1 + \sigma_2') \Psi_{2+}'^\mu(\tau + \sigma_2) + (1 - \sigma_2') \Psi_{2-}'^\mu(\tau - \sigma_2)}{\sqrt{2(1 - \sigma_2'^2) \langle \Psi_{2+}'^\mu(\tau + \sigma_2), \Psi_{2-}'^\mu(\tau - \sigma_2) \rangle}}. \end{aligned} \quad (21)$$

In last expression the continuity of the function $X^\mu(\tau, \sigma)$ and its tangential derivatives on the line $\sigma = \sigma_2(\tau)$ was taken into account. By differentiating on

τ the identity $X^\mu|_{\sigma=\sigma_2(\tau)-0} = X^\mu|_{\sigma=\sigma_2(\tau)+0}$ (after substitution (15)) we obtain

$$(1 + \sigma'_2) \Psi'_{1+}{}^\mu(+) + (1 - \sigma'_2) \Psi'_{1-}{}^\mu(-) = (1 + \sigma'_2) \Psi'_{2+}{}^\mu(+) + (1 - \sigma'_2) \Psi'_{2-}{}^\mu(-). \quad (22)$$

Here the notations $(+) \equiv (\tau + \sigma_2(\tau))$, $(-) \equiv (\tau - \sigma_2(\tau))$.

We transform the systems of the ordinary differential equations (18) - (21) to the normal form by the method used in Ref [10]. Let's consider transformation of the boundary condition (19) with the help given method.

Using the scalar product of the Eq. (21) by $(1 + \sigma'_2) \Psi'_{i+}{}^\mu(+) and $(1 - \sigma'_2) \Psi'_{i-}{}^\mu(-)$ and taking into account the isotropy of the vectors $\Psi'_{i\pm}{}^\mu$$

$$\Psi_{i+}^{\prime 2}(\tau) = \Psi_{i-}^{\prime 2}(\tau) = 0$$

[resulting from conditions (5)] we obtain the relations

$$\begin{aligned} \frac{1}{\sqrt{2}} \sqrt{(1 - \sigma_2^{\prime 2}) \langle \Psi'_{1+}{}^\mu(+), \Psi'_{1-}{}^\mu(-) \rangle} &= (1 + \sigma'_2) \langle U_2(\tau), \Psi'_{1+}{}^\mu(+) \rangle = \\ &= (1 - \sigma'_2) \langle U_2(\tau), \Psi'_{1-}{}^\mu(-) \rangle = \frac{1}{\sqrt{2}} \sqrt{(1 - \sigma_2^{\prime 2}) \langle \Psi'_{2+}{}^\mu(+), \Psi'_{2-}{}^\mu(-) \rangle} = \\ &= (1 + \sigma'_2) \langle U_2(\tau), \Psi'_{2+}{}^\mu(+) \rangle = (1 - \sigma'_2) \langle U_2(\tau), \Psi'_{2-}{}^\mu(-) \rangle. \end{aligned} \quad (23)$$

The equalities (21) and (23) result in the relations

$$(1 + \sigma'_2) \Psi'_{i+}{}^\mu(+) + (1 - \sigma'_2) \Psi'_{i-}{}^\mu(-) = 2(1 \pm \sigma'_2) \langle U_2(\tau), \Psi'_{j\pm}{}^\mu(\pm) \rangle U_2^\mu(\tau),$$

(right for all 8 variants of a choice of indexes i, j and signs \pm) with the help of which we shall transform the summands in Eq. (19) as follows:

$$\begin{aligned} (1 + \sigma'_2) \Psi'_{1+}{}^\mu(+) - (1 - \sigma'_2) \Psi'_{1-}{}^\mu(-) &= 2(1 - \sigma'_2) [\langle U_2, \Psi'_{1-}{}^\mu(-) \rangle U_2^\mu - \Psi'_{1-}{}^\mu(-)], \\ (1 - \sigma'_2) \Psi'_{2-}{}^\mu(-) - (1 + \sigma'_2) \Psi'_{2+}{}^\mu(+) &= 2(1 + \sigma'_2) [\langle U_2, \Psi'_{2+}{}^\mu(+) \rangle U_2^\mu - \Psi'_{2+}{}^\mu(+)]. \end{aligned}$$

The equation (19) take the normal form now

$$\frac{dU_2^\mu}{d\tau} = \frac{\gamma}{m_2} [\delta_\nu^\mu - U_2^\mu(\tau) U_{2\nu}(\tau)] [(1 + \sigma'_2) \Psi'_{2+}{}^\nu(+) + (1 - \sigma'_2) \Psi'_{1-}{}^\nu(-)], \quad (24)$$

where $\delta_\nu^\mu = \begin{cases} 1, & \mu = \nu \\ 0, & \mu \neq \nu. \end{cases}$

Using the same receptions we shall transform the equations (18) and (20) to the form

$$\frac{dU_1^\mu(\tau)}{d\tau} = \frac{\gamma}{m_1} [\delta_\nu^\mu - U_1^\mu(\tau) U_{1\nu}(\tau)] \Psi'_{1+}{}^\nu(\tau), \quad (25)$$

$$\frac{dU_3^\mu(\tau)}{d\tau} = \frac{\gamma}{m_3} [\delta_\nu^\mu - U_3^\mu(\tau) U_{3\nu}(\tau)] \Psi'_{2-}{}^\nu(\tau - \pi). \quad (26)$$

The systems (24) - (26) need the initial conditions

$$U_i^\mu(\tau(\lambda_i)) = v^\mu(\lambda_i) / \sqrt{v^2(\lambda_i)}, \quad i = 1, 2, 3. \quad (27)$$

Integrating systems (24) - (26) with the initial conditions (27) we can determine for $\tau > \tau(\lambda_i)$ unknown vector functions $U_i^\mu(\tau)$ with the help of the functions $\Psi_{i\pm}'^\mu$ known from the initial data on the segments (17). This procedure is limited on τ to ordinates of points in which the trajectories $\sigma = \sigma_i(\tau)$, $i = 1, 2, 3$ cross the characteristic lines $\tau \pm \sigma = \text{const}$ (Fig. 2). However we can continue this procedure for all τ if for every value of τ we determine the functions $\Psi_{i\pm}'^\mu$ outside segments (17) with the help of Eqs. (18) - (20), (24) - (26).

For this purpose we rewrite Eqs. (18) and (20) as

$$\Psi_{1-}'^\mu(\tau) = \Psi_{1+}'^\mu(\tau) - 2m_1\gamma^{-1}U_1'^\mu(\tau), \quad (28)$$

$$\Psi_{2+}'^\mu(\tau + \pi) = \Psi_{2-}'^\mu(\tau - \pi) - 2m_3\gamma^{-1}U_3'^\mu(\tau), \quad (29)$$

Eq. (28) corresponds to Eq. (25) and Eq. (29) — to Eq. (26). From Eqs. (19) and (22) we deduce two similar relations

$$\begin{aligned} \Psi_{1+}'^\mu(\tau + \sigma_2(\tau)) &= \Psi_{2+}'^\mu(\tau + \sigma_2(\tau)) - m_2[\gamma(1 + \sigma_2')]^{-1}U_2'^\mu(\tau), \\ \Psi_{2-}'^\mu(\tau - \sigma_2(\tau)) &= \Psi_{1-}'^\mu(\tau - \sigma_2(\tau)) - m_2[\gamma(1 - \sigma_2')]^{-1}U_2'^\mu(\tau). \end{aligned} \quad (30)$$

For solving the system (24) we are to determine the function $\sigma_2(\tau)$ for $\tau > \tau(\lambda_2)$. This function can be obtained by taking two of the equalities (23)

$$(1 - \sigma_2')\langle U_2(\tau), \Psi_{1-}'(-) \rangle = (1 + \sigma_2')\langle U_2(\tau), \Psi_{2+}'(+) \rangle$$

and expressing σ_2' as follows:

$$\frac{d\sigma_2(\tau)}{d\tau} = \frac{\langle U_2(\tau), [\Psi_{1-}'(\tau - \sigma_2) - \Psi_{2+}'(\tau + \sigma_2)] \rangle}{\langle U_2(\tau), [\Psi_{1-}'(\tau - \sigma_2) + \Psi_{2+}'(\tau + \sigma_2)] \rangle}. \quad (31)$$

The equations and systems (24) - (31) allow us to continue the functions $\Psi_{i\pm}'^\mu$ determined from the initial conditions with the help of formulas (16) on the segments (17). The constants of integration in $\Psi_{i\pm}'^\mu$ are determined from the initial condition (12). The described above algorithm of calculating $\Psi_{i\pm}'^\mu$ (with taking into account Eq. (15)) is the algorithm of solving the considered initial-boundary value problem with arbitrary initial conditions $\rho^\mu(\lambda)$, $v^\mu(\lambda)$.

3. Numerical solution of initial-boundary value problem

To the present time only two elementary motions of string configurations q-q-q are known. They are analytical solutions of equations (6) and satisfy

conditions (5), (7). One of these motions is spatially one-dimensional compression of a symmetric ($m_1 = m_3$, m_2 is at the center) rectilinear string that in some initial instant was at rest [5]. The second motion is a flat uniform rotation of the rectilinear string with the middle quark at rest at a center of rotation (in the reference frame of this center) [3, 8, 9]. If the choice of τ, σ satisfies the conditions (5) and (9) then this solution can be presented as

$$X^\mu = \omega^{-1} \{ a\tau; \sin(a\sigma - \phi_0) \cdot \cos a\tau; \sin(a\sigma - \phi_0) \cdot \sin a\tau \}. \quad (32)$$

Three coordinates of the vector X^μ are $X^0 = t$, $X^1 = x$, $X^2 = y$. The trajectory of the middle quark on a plane τ, σ has the form $\sigma_2(\tau) = \phi_0/a = \text{const}$ and the trajectories of the others quarks have the form (9).

The vector function (32) is the solution of the equations (6), satisfies the conditions (5) and under the relations

$$\frac{\gamma}{\omega m_1} = \frac{\sin \phi_0}{\cos^2 \phi_0}, \quad \frac{\gamma}{\omega m_3} = \frac{\sin(a\pi - \phi_0)}{\cos^2(a\pi - \phi_0)}, \quad \cos(a\sigma_i - \phi_0) > 0 \quad (33)$$

it satisfies the boundary conditions (7).

The solution (32) describes a rotation of the rectilinear string with an angular frequency ω . The quarks move at constant speeds

$$v_i = |\sin(a\sigma_i - \phi_0)|, \quad i = 1, 2, 3 \quad (34)$$

along the circles with the radii $R_i = v_i/\omega$. The relations (33) may be rewritten as

$$\frac{\gamma}{m_i} = \frac{\omega v_i}{1 - v_i^2} = \frac{v_i^2}{R_i(1 - v_i^2)}, \quad i = 1, 3. \quad (35)$$

The expression (32) can be obtained as the solution of the initial-boundary value problem (6), (7), (12), (13) with the initial conditions

$$\begin{aligned} \rho^\mu(\lambda) &= \{0; -R_1 + (R_1 + R_3)\lambda; 0\}, \\ v^\mu(\lambda) &= \{1; 0; -v_1 + (v_1 + v_3)\lambda\}, \end{aligned} \quad \lambda \in [0, 1], \quad \lambda_2 = \frac{R_1}{R_1 + R_3}. \quad (36)$$

If the parameters v_i, R_i satisfy Eqs. (35) the solution of the initial-boundary value problem has the form (32) in all domain Ω and can be obtained with the help of algorithm described in the previous section.

To solve the problem of stability of the solution (32) we are to consider an initial-boundary value problem with the initial conditions $\rho^\mu(\lambda), v^\mu(\lambda)$ which slightly differ from conditions (36). In particular, it is possible to put $\lambda_2 \neq R_1/(R_1 + R_3) = v_1/(v_1 + v_3)$, i.e. to place the middle quark not at the rotational center.

Figure 3: Motion of the system with initial conditions (36), (37). $\Delta t = 0.15$.

For arbitrary initial conditions the solution of the problem becomes complicated. In particular, system (24)–(26), (31) are solvable analytically only for exclusive cases. Therefore the use of numerical method is necessary. It makes possible to solve the initial-boundary value problem with any initial and boundary conditions.

The used below procedure of solving includes three stages:

1) realization of initial data, i.e., the determination the functions $\Psi'_{i\pm}{}^\mu$ in the segments (17) by the formulas (16) [with the help the initial conditions $\rho^\mu(\lambda)$ $v^\mu(\lambda)$].

2) continuing the functions $\Psi'_{i\pm}{}^\mu(\xi)$ (with integrating) with the help of boundary conditions (24)–(31);

3) searching of level lines of the world surface $t = X^0(\tau, \sigma) = \text{const}$ and drawing of their projections on the plane Oxy ($x \equiv X^1$, $y \equiv X^2$).

The last item is required for the best representation of the string motion. We show “photographs” of its sequential positions made at regular intervals Δt .

The results of calculations for the q-q-q system with $m_1 = 1$, $m_2 = 3$, $m_3 = 2$, $\gamma = 1$ obtained with use of the MATLAB package are shown in Fig. 3.

The initial conditions for the given motion have form close to (36), where

$$\omega = 2, \quad v_1 \simeq 0.4142, \quad v_3 \simeq 0.4142, \quad \lambda_2 = 0.615 \neq v_1/(v_1 + v_3) \simeq 0.637. \quad (37)$$

The values R_1 , v_1 satisfy conditions (35) however the middle quark is located not at the center of rotation.

In Figs. 3a–d on the plane Oxy the positions of the system are shown sequentially with the interval $\Delta t = 0.15$ in time. The initial position of the string (36) in Fig. 3 is marked by number 1 and next (with growth of t) “photographs” of the system are marked by numbers 2, 4, ..., near the third quark position. For convenience of perception the first pointlike quark q_1 with mass $m_1 = 1$ at the end of the string is depicted as a green square, the third quark q_3 with mass $m_3 = 2$ — as a red square, and positions of the middle quark q_2 are marked by blue circles.

The last position of the system number 11 in Fig. 3 is the first one in Fig. 3b. The system q-q-q continues rotating as a whole in the counter-clockwise direction in Figs. 3b–d. At the beginning (Fig. 3) the motion of the system as a whole is close to the uniform rotation (32). Later the middle quark q_2 under the centrifugal force is displaced to the quark q_1 but there is no junction of q_2 with q_1 and transformation of the system in the quark-diquark one [3]. The

middle quark plays a role of rotational center for the segment of the string $q_2 - q_1$ (positions 26, 27 in Fig. 3c) and then it returns to the center of a system (Fig. 3c, 3d). The middle quark does not reach this center and begins again centrifugal moving away with recurring of all cycle.

As we see, the motion is quasiperiodic. It's qualitative features are the same for rather wide variation of parameters of the initial-boundary value problems. In particular, decreasing of the initial deviation of q_2 from the center of masses reduces in retarding of the initial stage of motion.

If the initial conditions differ from (36) by a small nonsymmetrical correction to the velocity $v^\mu(\lambda)$ (for $\lambda_2 = \frac{R_1}{R_1+R_3}$), the motion of the system is approximately the same as in Fig. 3. The increase of the mass m_2 of the middle quark q_2 reduces in Increase of ΔR — minimal length of the segment $q_1 - q_2$.

Energy or velocity of rotation of the system under the relations (35) weakly influences to the character of motion. In the non-relativistic limit $v_i \ll 1$ and in relativistic case (Fig. 3) the period of cyclical motion of the middle quark between the center and the end is approximately equal to the rotational time of the string.

Conclusion

In the present work the method of solution of the initial-boundary value problem for classical motion of the linear string baryon model q-q-q is suggested. This approach allows us to solve the stability problem for the known solution (32) — flat uniform rotation of the rectilinear string with the quark q_2 at rest. This solution is unstable in Lyapunov's sense — the as much as small perturbations result in complicated motion (Fig. 3) with quasi-periodical varying of the distance between the nearest two quarks. However the minimal value of the mentioned distance ΔR does not equal zero, in other words, the system q-q-q is not transformed in quark-diquark (q-qq) one, as was supposed speculatively [3].

The analysis of this problem does not permit us to exclude the linear configuration q-q-q from a set of string baryon configurations (q-qq, Y, Δ) for describing baryonic orbital excitations on the Regge trajectories. At that it is possible to use expressions for the total energy and the angular moment of the system q-q-q, obtained in Ref. [8] for the motion (32).

$$E = \sum_{i=1}^3 \left[\frac{\gamma}{\omega} \arcsin v_i + \frac{m_i}{\sqrt{1-v_i^2}} \right] + \Delta E_S,$$

$$J = \sum_{i=1}^3 \left[\frac{1}{2\omega} \left(\frac{\gamma}{\omega} \arcsin v_i + \frac{m_i v_i^2}{\sqrt{1-v_i^2}} \right) + s_i \right].$$

Here ΔE_S is the spin-orbital correction to the energy, s_i are projections of quark spins, $v_2 = 0$. These expressions are applicable and for more complicated motions, obtained with the help of weak perturbations of the conditions (36) (see Fig. 3) because of conservation of E and J . Hence, for perturbed quasiperiodic motions the graph of dependence $J = J(E^2)$ is close to linear, that allows us to describe baryonic Regge trajectories with the help of the model q-q-q [8].

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